

The retailer's optimal order policy with inventory-level-dependent demand under trade credit limit

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Abstract

This paper incorporates the consideration of trade credit limit (TCL) when addressing the retailer's optimal operational policy for the item with inventory-level-dependent demand. That is, if the retailer's purchase cost is below TCL, the supplier offers the retailer a full delay in payments; otherwise, the retailer is allowed to delay payment for the amount up to TCL. By formulating the retailer's average profit function, the retailer's optimal order quantity can be obtained. Using numerical analyses, the retailer's order quantity and the corresponding account payable can be effectively controlled under a given predetermined trade credit limit.

Keywords: Inventory-level-dependent demand; Trade credit limit; Order policy

Introduction

Trade credit policy has been an increasing major role in short-term financing in the retail sector. To be specific, the supplier grants the retailer credit period rather than demanding immediate cash payment to encourage larger order quantities from the retailer. However, as indicated by (Iglesias et al., 2007), trade credit can be considered as a means for the retailer to obtain low-cost financial resources from the supplier. That is, under the fixed credit period, the retailer's increased order quantities result in higher value of the retailer's account payable, and thus the supplier would pay off more opportunity cost of capital. Practically, the supplier would usually set trade credit limit (TCL) to optimally cap retailers' account payable (Cai et al., 2014). (Seifert et al., 2013) also perceive a limit set of credit terms as a sufficient solution when dealing with the retailer's default risk.

(Balakrishnan et al., 2004) argue that in some retail contexts, such as groceries, apparel and bookstores, stocking large quantities of a popular product can be an effective tool for the retailer to stimulate market demand, characterized as inventory-level-dependent demand (ILDD). Under ILDD, inventory plays both promotional and

service roles, and the retailer could prompt the “stack them high, let them fly” phenomenon by keeping high inventory level. However, in the view of the supplier, when offering the retailer credit period for the item with ILDD, the retailer’s increased order quantity derived from the joint effects of credit period and ILDD would reduce the supplier’s working capital. Consequently, the supplier would not only need more working capital, but also sacrifice more opportunity cost of capital.

To sum up, under ILDD, this paper first analyzes the effects of trade credit limit towards the retailer’s optimal order quantity. In this paper, the supplier’s credit term can be interpreted as follows. When the retailer’s purchase cost is less than the predetermined trade credit limit, the supplier offers the retailer full delay in payments, otherwise, and the retailer is allowed to delay payment for the amount up to the trade credit limit while the excess part has to be paid off immediately.

Literature review

Generally, trade credit policy granted by the supplier can be regarded as an incentive mechanism to stimulate the retailer’s order quantity. A growing number of researchers have already studied the retailer’s optimal order policy under the fixed credit period under various circumstances, such as delay in payments linked to order quantity (Chang et al., 2009), deteriorating item (Yang et al., 2015), capacity constraint (Ouyang et al., 2015), partial delay in payments (Yong Wu Zhou et al., 2015). Confronted with negative effects of the occupation risk of the supplier’s working capital, the supplier usually sets a trade credit limit to manage the capital risk when offering the retailer credit period. With the assumption of limit credit, (Burkart & Ellingsen, 2004) developed a model to verify that trade credit and bank credit can be either complements or substitutes. (Cai et al., 2014) discussed the creditors’ optimal trade credit limits when a capital-constrained retailer faces demand uncertainty. Under the given retailer’s upper limit of account payable, (Jia et al., 2016) proposed an economic order quantity model with inventory-level-dependent demand, which aims to explore its influences on the retailer’s optimal order policy.

Recognizing the importance of ILDD, research efforts have already devoted to analyzing its influences on the operational strategies. Generally, as Urban (2005) noted, the demand-stimulating effect of inventory can fall under two categories in the literatures, i.e. initial-inventory-level dependent (Type I) and instantaneous-inventory-level dependent (Type II). Many papers have already considered ILDD under trade credit policy. (Y. W. Zhou et al., 2012) studied a two-echelon supply chain for items with trade credit, which faces an inventory-dependent demand and limited displayed-shelf space. (Soni, 2013) analyzed the retailer’s replenishment policy for non-instantaneous deteriorating items with demand sensitive to both price and inventory quantity under permissible delay in payment. (Mo et al., 2014) developed an optimal order policy for perishable multi-item inventory under inventory dependent demand and two-level trade credit. However, under trade credit policy, the above papers ignore negative impacts of ILDD on the supplier’s working capital (i.e. the increase in retailer’s account payable), which is worthy of further investigation.

Model formulation

The single-retailer inventory system involves a single item with ILDD, and the retailer’s order quantity could be completed instantaneously. Thus, the variation of the inventory level $I(t)$ within the time interval $[0, T]$ can be described by the following differential

equation: $\frac{dI(t)}{dt} = -\alpha I(t)^\beta$, $0 \leq t \leq T$. Considering the boundary condition $I(0) = Q$, the solution of the above equation is

$$I(t) = [Q^{1-\beta} - \alpha t(1-\beta)]^{\frac{1}{1-\beta}}, \quad 0 \leq t \leq T. \quad (1)$$

Referring to (Y. W. Zhou et al., 2012), the demand rate of item $D(t)$ is a known function of the retailer's instantaneous inventory level $I(t)$, i.e., $D(t) = \alpha[I(t)]^\beta$, where scale parameter $\alpha > 0$ and shape parameter $0 < \beta < 1$. Correspondingly, the demand rate can be obtained as follows:

$$D(t) = \alpha [Q^{1-\beta} - \alpha t(1-\beta)]^{\frac{\beta}{1-\beta}}, \quad 0 \leq t \leq T. \quad (2)$$

By substituting $t = T$ into Eq. (1) leads to $T = \frac{Q^{1-\beta}}{\alpha(1-\beta)}$. Under the circumstance, by setting $T = T_w$, we can obtain $T_w = \frac{Q_w^{1-\beta}}{\alpha(1-\beta)}$. Similarly, by letting $T = M$ and $T = T_w + M$, respectively, we can obtain $Q_M = [\alpha(1-\beta)M]^{\frac{1}{1-\beta}}$, and $Q_{Mw} = [\alpha(1-\beta)(T_w + M)]^{\frac{1}{1-\beta}}$.

Since trade credit limit W is tightly associated with the scales, assets and liabilities of the retailer, it should be predetermined by the supplier before the transaction. Thus, under the given trade credit limit, W , the trade credit policy can be interpreted as the following two cases.

- (i) Full delay in payments (FDP) case: $Q \leq Q_w$ (i.e., $T \leq T_w$). That is, when the retailer's account payable $c_w Q$ is less than W , the supplier offers the retailer fully delay in payments of M period.
- (ii) Partial delay in payments (PDP) case: $Q_w \leq Q$ (i.e., $T_w \leq T$). That is, when the retailer's account payable $c_w Q$ is more than W , the retailer is allowed to delay payment W while the excess purchases $c_w(Q - Q_w)$ should be paid off immediately.

Under the trade credit limit policy, there are two cases to figure out the retailer's opportunity cost of capital per unit time, including FDP case and PDP case.

FDP case. $Q \leq Q_w$

In FDP case, the retailer can get the interest earned from sales revenue during the time interval $[0, M]$. And referring to (Zhong & Zhou, 2013), in terms of the interest charged by the retailer, there are two subcases based on the relationship between Q and Q_M (i.e., T and M).

- (i) $Q \leq Q_M$ (i.e., $T \leq M$): there is no interest charged on items in stock for the retailer. Thus, the retailer's opportunity cost of capital per unit time = $\frac{c_w I_e}{T} \left[\int_0^T \int_0^t D(u) du dt + (M - T)Q \right]$.
- (ii) $Q_M \leq Q$ (i.e., $M \leq T$): the retailer should pay the interest charged on items in stock during the time interval $[M, T]$. Thus, the retailer's opportunity cost of capital per unit time = $\frac{c_w I_e}{T} \int_0^M \int_0^t D(u) du dt - \frac{c_w I_c}{T} \int_M^T I(t) dt$.

With the assumption of $I_e = I_c$, we can easily find the expression in (i) $Q \leq Q_M$ is the same as the expression in (ii) $Q_M \leq Q$. Thus, in FDP case, the retailer's opportunity cost of capital per unit time can be summarized as $= c_w I_e (1 - \beta) \left(\alpha M Q^\beta - \frac{Q}{2 - \beta} \right)$.

PDP case. $Q_w \leq Q \leq Q_{Mw}$

Under the circumstance, there exists a time point T_0 satisfying the following expression: $I(T_0) = Q_w$, where $T_0 \in [0, T]$. By substituting $I(T_0) = Q_w$ into Eq. (1) and comparing with $T_w = \frac{Q_w^{1-\beta}}{\alpha(1-\beta)}$, we have $T_0 = T - T_w$. Thus, T_0 indicates the time interval when the retailer's inventory level decreases from Q to the threshold order quantity, Q_w . In PDP case, the retailer can earn the interest from the sales revenue within credit period, M . And there are two types of interests charged by the retailer. Specifically, the length of M determines the interest charged for the item still in stock after credit period (denoted as IM), while the value of T_0 (i.e. $T - T_w$) influences the interest charged on the item due to the immediate payment (denoted as IT). As mentioned above, in PDP case, the assumption of $T \leq T_w + M$ is adopted to avoid the discussion of an overlap between IM and IT. Considering the relation between M and T , two situations should be further classified.

- (i) $Q \leq Q_M$ (i.e., $T \leq M$): the retailer should be charged the interest on the item of immediate payment during the time interval $[0, T_0]$. Thus, the opportunity cost of capital per unit time $= \frac{c_w I_e}{T} \left[\int_0^T \int_0^t D(u) du dt + (M - T)Q \right] - \frac{c_w I_c}{T} \int_0^{T_0} (I(t) - Q_w) dt$.
- (ii) $Q_M \leq Q \leq Q_{Mw}$ (i.e., $M \leq T \leq T_w + M$): the retailer should be charged the interest on the item of immediate payment during the time interval $[0, T_0]$ and pay the interest for the item of delayed payment still in stock from time M to time T , and thus the opportunity cost of capital per unit time $= \frac{c_w I_e}{T} \int_0^M \int_0^t D(u) du dt - \frac{c_w I_c}{T} \left[\int_0^{T_0} (I(t) - Q_w) dt + \int_M^T I(t) dt \right]$.

Since $I_e = I_c$, we can easily find that the expression in (i) $Q \leq Q_M$ is the same as the expression in (ii) $Q_M \leq Q \leq Q_{Mw}$. Thus, in this subcase, the retailer's opportunity cost of capital per unit time $= c_w I_e \left[\alpha M Q^\beta (1 - \beta) + Q_w - \frac{2Q(1-\beta) + Q_w^{2-\beta} Q^{\beta-1}}{2-\beta} \right]$.

Considering other elements of the retailer's profit function: (1) the setup cost per unit time $= \frac{K_r \alpha (1-\beta)}{Q^{1-\beta}}$; (2) the holding cost per unit time (excluding interest charges) $= \frac{h_r Q (1-\beta)}{2-\beta}$ and (3) the selling profit per unit time $= \alpha Q^\beta (1 - \beta) (p - c_w)$. Thus, the retailer's average profit per unit time can be presented as follow:

$$\Pi_r(Q) = \begin{cases} \Pi_{r1}(Q), & Q \leq Q_w, \\ \Pi_{r2}(Q), & Q_w < Q \leq Q_{Mw}, \end{cases} \quad (3)$$

where

$$\Pi_{r1}(Q) = (1 - \beta) \left[\alpha Q^\beta (p - c_w + c_w I_e M) - \frac{K_r \alpha}{Q^{1-\beta}} - \frac{(h_r + c_w I_e) Q}{2 - \beta} \right], \quad (4)$$

$$\begin{aligned} \Pi_{r_2}(Q) = (1 - \beta) & \left[\alpha Q^\beta (p - c_w + c_w I_e M) - \frac{K_r \alpha}{Q^{1-\beta}} - \frac{(h_r + 2c_w I_e) Q}{2-\beta} \right] \\ & - c_w I_e \left(\frac{Q_w^{2-\beta} Q^{\beta-1}}{2-\beta} - Q_w \right). \end{aligned} \quad (5)$$

where $\Pi_{r_1}(Q_w) = \Pi_{r_2}(Q_w)$, $\Pi_r(Q)$ is continuous and well defined.

The analyses of the retailer's optimal solution

In this section, we present the decision process for the under the trade credit limit, W . Under the value of M granted by the supplier, the retailer aims to maximize the average profit by determining the optimal order quantity, Q^{opt} . Thus, by taking the first-order derivative of $\Pi_{r_1}(Q)$ in Eq. (4) and $\Pi_{r_2}(Q)$ in Eq. (5) with respect to Q , respectively, we can obtain

$$f_1(Q) = (1 - \beta) \left[\alpha \beta Q^{\beta-1} (p - c_w + c_w I_e M) + \alpha K_r Q^{\beta-2} (1 - \beta) - \frac{h_r + c_w I_e}{2-\beta} \right], \quad (6)$$

$$\begin{aligned} f_2(Q) = (1 - \beta) & \left[\alpha \beta Q^{\beta-1} (p - c_w + c_w I_e M) + \alpha K_r Q^{\beta-2} (1 - \beta) \right. \\ & \left. - \frac{h_r + 2c_w I_e}{2-\beta} + \frac{c_w I_e Q_w^{2-\beta}}{(2-\beta) Q^{2-\beta}} \right]. \end{aligned} \quad (7)$$

Via analyzing the properties of $f_i(Q)$, $i = 1, 2$, the following theoretical result can be used to obtain the optimal value of $\Pi_{r_i}(Q)$, $i = 1, 2$.

Lemma 1. The retailer's optimal order quantity is given as follows.

Cases	Situations	Q^*	$\Pi_r(Q^*)$
FDP	$f_1(Q_w) \leq 0$	Q_a	$\Pi_{r_1}(Q_a)$
	$f_1(Q_w) > 0$	Q_w	$\Pi_{r_1}(Q_w)$
PDP	$f_2(Q_w) \leq 0$	Q_w	$\Pi_{r_2}(Q_w)$
	$f_2(Q_w) > 0$ and $f_2(Q_{Mw}) \leq 0$	Q_b	$\Pi_{r_2}(Q_b)$
	$f_2(Q_w) > 0$ and $f_2(Q_{Mw}) > 0$	Q_{Mw}	$\Pi_{r_2}(Q_{Mw})$

Note: it should be mentioned that $f_1(Q_a) = 0$ and $f_2(Q_b) = 0$.

Proof:

(i) By taking the first order derivative of $f_1(Q)$ in Eq. (6), we have

$$\frac{df_1(Q)}{dQ} = -\alpha(1 - \beta)^2 Q^{\beta-2} \left[\beta(I_e c_w M + p - w) + \frac{K_r(2-\beta)}{Q} \right] < 0. \quad (8)$$

Thus, $\Pi_{r_1}(Q)$ is concave for $T \in (0, +\infty)$ and there exists a unique solution (say, Q_a) satisfying the equation of $f_1(Q_a) = 0$. From Eq. (6), it is apparent that $Q_a > 0$. And by analyzing the sign of the boundary value $f_1(Q_w)$, the following situations can be obtained.

- (a) If $f_1(Q_w) \leq 0$, $\Pi_{r_1}(Q)$ has a maximum value at the point $Q_1 = Q_a$, where $f_1(Q_a) = 0$.
 - (b) If $f_1(Q_w) > 0$, $\Pi_{r_1}(Q)$ has a maximum value at the point $Q_1 = Q_w$.
- (ii) By taking the first order derivative of $f_2(Q)$ in Eq. (7), we have

$$\frac{df_2(Q)}{dQ} = -\alpha(1-\beta)^2 Q^{\beta-2} \left[\beta(I_e c_w M + p - w) + \frac{K_r(2-\beta)}{Q} \right] - \frac{(1-\beta)I_e c_w Q_w^{2-\beta}}{Q^{3-\beta}} < 0. \quad (9)$$

Thus, $\Pi_{r2}(Q)$ is concave for $T \in (0, +\infty)$ and there exists a unique solution (say Q_b) satisfying the equation of $f_2(Q_b) = 0$. And by analyzing the signs of the boundary value $f_2(Q_w)$ and $f_2(Q_{Mw})$, the following situations can be obtained.

- (a) If $f_2(Q_w) \leq 0$, $\Pi_{r2}(Q)$ has a maximum value at the point $Q_2 = Q_w$.
- (b) If $f_2(Q_w) > 0$ and $f_2(Q_{Mw}) \leq 0$, $\Pi_{r2}(Q)$ has a maximum value at the point $Q_2 = Q_b$, where $f_2(Q_b) = 0$.
- (c) If $f_2(Q_w) > 0$ and $f_2(Q_{Mw}) > 0$, $\Pi_{r2}(Q)$ has a maximum value at the point $Q_2 = Q_{Mw}$.

To conclude, let $\Delta_1 = f_1(Q_w) = f_2(Q_w)$ and $\Delta_2 = f_2(Q_{Mw})$; thus, we have $\Delta_1 > \Delta_2$. By means of Lemma 1 and multiple combinations of Δ_1 & Δ_2 , the following Theorem 1 can be obtained to determine the retailer's optimal order quantity Q^{opt} .

Theorem 1

- (1) If $\Delta_1 < 0$ and $\Delta_2 < 0$, we have $Q^{opt} = Q_a$.
- (2) If $\Delta_1 \geq 0$ and $\Delta_2 < 0$, we have $Q^{opt} = Q_b$.
- (3) If $\Delta_2 \geq 0$, we have $Q^{opt} = Q_{Mw}$.

From Theorem 1, Q_w influences the retailer's preference towards the credit terms, which induce the retailer to adopt FDP case or PDP case. To further analyze the impact of the supplier's credit period towards the retailer's order policy, we aim to verify the dependence between Q^{opt} and M , as depicted in the following corollary.

Corollary 2:

Both Q_a and Q_b are positively associated with credit period M .

Proof:

By taking the first-order derivative of $f_1(Q_a) = 0$ and $f_2(Q_b) = 0$ with respect with M , respectively, we can obtain $\frac{dQ_a}{dM} = \frac{\beta c_w I_e Q_a^2}{\beta Q_a (1-\beta)(p-c_w+c_w I_e M) + K_r(1-\beta)(2-\beta)}$, and $\frac{dQ_b}{dM} = \frac{\alpha \beta c_w I_e Q_b^2}{\alpha \beta Q_b (1-\beta)(p-c_w+c_w I_e M) + \alpha K_r(1-\beta)(2-\beta) + c_w I_e Q_w^{2-\beta}}$. It is easy to find $\frac{dQ_a}{dM} > 0$ and $\frac{dQ_b}{dM} > 0$, and thus both Q_a and Q_b are positively associated with the length of credit period M .

Numerical examples

In this section, the values of parameters are set as follows: $K_r = 80$, $I_e = 0.3$, $I_c = 0.3$, $h_r = 2$, $\alpha = 26$, $\beta = 0.3$, $p = 24$, $c_w = 18$, $W = 1500$, $M = 1$. In this example, we change the values of trade credit limit, W and credit period, M , respectively, which aims to measure their effects towards the optimal solutions of the supply chain. And the results can be shown in Table 1.

Table 1 The optimal solutions under different values of W and M

Parameters		Q^{opt}	Π_r	AR^{opt}
W	1700	93.69	463.85	1686.42
	1600	88.89	463.52	1600
	1500	83.33	647.47	1500
	1400	77.78	459.90	1400
	1300	72.22	456.31	1300
M	1.5	83.33	647.47	1500
	1	83.33	462.25	1500
	0.75	82.96	369.64	1493.33
	0.5	73.07	278.89	1315.34
	0.25	64.06	191.58	1153.12

Note: AR^{opt} denotes the retailer's account payable during the credit period, where $AR^{opt} = c_w Q$ for FDP case and $AR^{opt} = W$ for PDP case.

According to the above results, conclusions can be drawn as follows:

- (i) Along with the decrease of trade credit limit, W , the retailer prefers to reduce Q^{opt} , which verifies that lower values of W may restrain the retailer's order quantity to some extent. And consequently, the retailer's account payable AR^{opt} could be completely controlled within the predetermined trade credit limit, implying the positive effects of the credit term towards the supplier's capital risk.
- (ii) When confronted with higher values of M , it is apparent that Q^{opt} , Π_r and AR^{opt} increase. That is, stimulated by the longer credit period, the retailer is willing to have higher order quantities in order to reap the benefits of the opportunity cost of the capital, though the retailer's account payable also increases to some degree accordingly.

Conclusions

Under the predetermined trade credit limit, W and credit period, M , the retailer's inventory system is developed for the item with inventory-level dependent demand in order to reconsider the retailer's optimal order quantity. That is, when the retailer's order quantity Q is less (more) than the threshold order quantity Q_w , FDP (PDP) case would be granted to the retailer by the supplier. Through redefining the opportunity cost of capital of the supplier and the retailer, the corresponding average profit functions are derived, and then the optimal solutions of FDP case and PDP case can be developed, respectively. According to the theoretical results and numerical examples, it can be found that under a given trade credit policy, it is effective for the supplier to apply trade credit limit to the retailer in order to mitigate working capital risk, and thus the retailer's account payable, AR^{opt} could be completely controlled below the predetermined trade credit limit.

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