Investigating an imbalanced consideration of the supply line from a control theory perspective

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Abstract
This paper addresses the well-established finding that individuals in supply chains often do not base their order decisions to the same degree on information they have about the supply line as on information they have about their stock. The accompanying advantages and disadvantages of this phenomenon are investigated by considering insights from production and inventory control theory. A system dynamics model representing the Beer Distribution Game was used for analysis. Results indicate that an imbalanced consideration of the supply line can be harmful in some cases but also beneficial in other cases.

Keywords: Order Heuristics, Control Theory, Beer Distribution Game

Introduction
One of the crucial objectives of Supply Chain Management is to maintain a high customer service level while simultaneously avoiding excess inventory throughout the supply chain (Mentzer et al., 2001). An opportunity to experience the difficulty of that seemingly easy task, is provided by the Beer Distribution Game, where participants need to manage a simulated serial supply chain and regularly have a hard time to handle the trade-off between having sufficient but not unnecessary high inventory levels (Sterman, 1989). A commonly observable phenomenon during the Beer Distribution Game is an increase in demand variability from downstream to upstream stages, also known as the bullwhip effect, leading to avoidable excess stock throughout the supply chain (Lee et al., 1997). In a seminal paper, Lee et al. (1997) deduced four important causes of the bullwhip effect: order batching, price fluctuations, demand forecasting and shortage gaming. The traditional Beer Distribution Game eliminates three of these four causes by (1) avoiding fixed ordering costs (no need for order batching), (2) removing the option to withhold products (no need for shortage gaming) and (3) removing any pricing opportunities (no price fluctuations). Modified versions of the Beer Distribution Game also control for demand forecasting by introducing a stationary demand which is known to everyone in the supply chain (Croson & Donohue, 2003, 2006). Despite the elimination of the four causes identified by Lee et al. (1997), a stream of research showed the persistence of the

bullwhip effect mainly due to behavioral causes (Croson & Donohue, 2003, 2006; Croson et al., 2014).

One of the behavioral causes which Niranjan et al. (2009) consider as "the cornerstone of the theory of the behavioral causes of the bullwhip effect" (p. 355) is called Supply Line Underweighting. Supply Line Underweighting is based on the observation by Sterman (1989) that individuals' order decisions follow an anchoring and adjustment heuristic. According to this observation, decision makers anchor their order quantity on the expected demand which needs to be satisfied and subsequently adjust the order rate by two fractions. One of these fractions accounts for the discrepancy between desired and actual stock level, the other fraction accounts for the discrepancy between desired and actual supply line level. Statistical analysis revealed the tendency of individuals to base the adjustment of their order decisions to a higher extent on the discrepancy between desired and actual stock level than on the discrepancy between desired and actual supply line level, which can be interpreted as and underweighting of the supply line (Croson et al., 2014; Sterman, 1989). The concept behind the order heuristic which leads to Supply Line Underweighting is also part of a body of literature that deals with production and inventory control theory (Naim & Towill, 1995).

This paper applies findings from the field of production and inventory control theory for the investigation of Supply Line Underweighting in the context of the Beer Distribution Game. A system dynamics simulation shows that considering both fractions to the same amount is more cost efficient than Supply Line Underweighting when it comes to the performance of the supply chain in the Beer Distribution Game. Furthermore, the study on the one hand demonstrates that cost-efficiency can be increased even more by making use of Supply Line Overweighting, which is essentially the counterpart of Supply Line Underweighting. On the other hand, the study also points out the substantial risks which are associated with Supply Line Overweighting.

Supply Line Underweighting framed in production and inventory control theory

Control theory deals with the study of dynamic systems and was traditionally mostly applied in engineering. In the 1950s, amongst others propelled by Simon (1952) and Vassian (1955), the application of control theory to the field of production and inventory control gained momentum. Three decades later, Towill (1982) introduced the so called Inventory and Order Based Production Control Systems (IOBPCS), which is based on a periodic review policy for placing orders in a supply line. In case of a basic IOBPCS, orders are solely based on incoming demand from a customer and the discrepancy between desired and actual inventory, whereas the desired inventory level is fixed. John et al. (1994) further developed this model to an Automatic Pipeline Inventory and Order Based Production Control System (APIOBPCS) by incorporating an additional feedback loop, which considers the discrepancy between the desired and actual supply line level. The APIOBPCS archetype assumes a fixed inventory target level, but a variable supply line target level. The fixed inventory target level needs to be carefully defined in advance, whereas the variable supply line target is calculated by multiplying the forecasted expected loss rate with the process acquisition lag (John et al., 1994). The APIOBPCS policy is equivalent to the order heuristic observed by Sterman (1989) in the Beer Distribution Game (John et al., 1994; Naim & Towill, 1995). The order heuristic as well as the APIOBPCS model is described by Equation 1.

Equation (1):

$$IO = \hat{L} + \alpha_S(S^* - S) + \alpha_{SL}(SL^* - SL) \qquad IO \geq 0$$

According to Equation 1, decision makers base their order quantities (IO) on an expected demand ($\hat{L}$) and adjust this quantity by a fraction (α_S) of the discrepancy between the desired and actual inventory level ($S^* - S$) as well as a fraction (α_{SL}) of the difference between a target and the actual supply line level ($SL^* - SL$) (Sterman, 1989). In control theory, the variables representing the fractions (α_S and α_{SL}) are also called proportional controllers. Selecting the appropriate values for the proportional controllers throughout the supply chain is critical for its ability to satisfy customer demand while keeping inventory levels low. If both proportional controllers are set to one (i.e., α_S and $\alpha_{SL} = 1$), the full discrepancies are recovered every period. In case of matching proportional controllers of a value smaller than 1, a smoothing replenishment policy is applied. (Disney et al., 2007). Such a smoothing replenishment policy reduces the bullwhip effect (Dejonckheere et al., 2004) and limits inventory costs (Chen & Disney, 2007) but can also increase net inventory variance and consequently increases the chance of not being able to fulfill customer demand (Disney et al., 2007; 2006).

Empirical data collected from many runs of the Beer Distribution Game show the difficulty for human players to tune the proportional controllers to optimal values. Recalling the anchoring and adjustment heuristic by Sterman (1989), people tend to systematically underweight the supply line in that, they choose a lower value for the proportional controller concerning the supply line than for the one controlling the inventory level. This results in large fluctuations of orders, supply line levels and eventually inventories which are amplified as one moves upstream the supply chain. For example, if demand increases and the supply line is underweighted, the actual stock level will be, after a delay, higher than the desired stock level. This in turn triggers periods of underordering (Riddalls & Bennett, 2002).

Setting both proportional controllers to the same value implies absence of Supply Line Underweighting. This case of equal proportional controllers was studied by Deziel & Eilon (1967) and later classified as a special case of APIOBPCS named DE-APIOBPCS after the first letters of each author’s name (Disney & Towill, 2003). Following the DE-APIOBPCS policy, it is guaranteed that the system is stable for arbitrary lead times and that oscillations in the order rate are avoided (Disney & Towill, 2002; Disney et al., 2004). Stability refers to the ability of a system to return to a steady-state after an interference whereas an unstable system will oscillate with a successively growing amplitude. A stable system can be further broken down into overdamped, underdamped and critically damped systems. An overdamped system reaches its steady-state without oscillations but slower than a critically damped system, which reaches the steady-state as quickly as possible. An underdamped system however reaches its steady-state only after oscillating for a while with a gradually decreasing amplitude (Aström & Murray, 2010). Also, unequal proportional controllers can lead to a stable system. Riddalls & Bennett (2002) state that the APIOPBCS model remains stable irrespective of the delay time as long as α_{SL} / α_s > 0.5. This finding shows that various combinations of proportional controller values can lead to a stable system although the costs incurred may differ between different combinations of proportional controllers.

Studies have shown that Supply Line Underweighting can result in a bullwhip effect and may cause underdamped or locally unstable systems (Croson et al., 2014; Mosekilde & Laugesen, 2007). The DE-APIOBPCS policy not only guarantees a stable system, it also prevents oscillations in the order rate (Disney et al., 2004). This indicates either an overdamped or critically damped system. Oscillations in distribution chains are undesirable because they can lead to fluctuating inventory levels and periods with a higher stock level than desired but also a higher risk of stockouts (Riddalls & Bennett, 2002).

The performance in the Beer Distribution Game is measured by two types of costs: inventory and backlog costs. The characteristics of the DE-APIOBPCS policy to produce a stable system while avoiding oscillations leads to the hypotheses that matched proportional controllers ($\alpha_{SL} = \alpha_S$) result in a better performance, indicated by lower costs compared to a policy of Supply Line Underweighting ($\alpha_{SL} < \alpha_S$) in the Beer Distribution Game setting.

Hypothesis 1: Given the restriction $\alpha_{SL} \leq \alpha_S$, equally weighted proportional controllers result in the lowest accumulated costs over all stages in the Beer Distribution Game.

Hypothesis 1 considers only cases of Supply Line Underweighting and equally weighted proportional controllers. As mentioned before, not only equal, but also unequal proportional controllers can lead to a stable system. The before mentioned stability criteria can also be met in case of $\alpha_{SL} > \alpha_S$, which can be called Supply Line Overweighting. Consequently, it seems possible that there is a combination of proportional controllers which leads to a system that not only meets the stability criteria, but also incurs less costs compared to a system with equally weighted proportional controllers.

Hypothesis 2: There is a constellation of the proportional controllers, which indicates Supply Line Overweighting ($\alpha_{SL} > \alpha_S$) and results in lower costs than the constellation of the optimally equal-weighted proportional controllers.

Methodology and model structure

A system dynamics model was used to test the hypotheses. This approach has been selected because of the high order of the non-linear difference equations (23rd order) which makes analytical analysis intractable (Sterman, 1989). Furthermore, the method makes the underlying dynamics of the system visible and ready for investigation. To run the simulations and generate valid data, a system dynamics models is needed which represents the Beer Distribution Game accordingly.

The Beer Distribution Game reflects a distribution chain where four human players are assigned to stages representing a retailer, a distributor, a wholesaler and a factory. The players' objective is to satisfy demand requests from an external customer to the lowest costs possible, whereby the customer's demand is simulated by a deck of cards which -in the standard game set up- follows a pattern where demand is 4 units for the first 4 periods and steps up to 8 units in period 5 where it remains for the rest of the game. Order information flows upstream from the retailer to the factory and products flow downstream from the factory to the retailer. Until order information from one stage arrives at the subsequent stage, two time periods pass. The same delay of two time periods exists for the flow of products from one stage to the next one downstream. Both kinds of delay taken together result in a four-period delay from placing to receiving an order. The only exception is the factory, which gets its ordered products already after three periods. Costs are caused by holding inventory (i.e., \$0.50 per case of beer per week) as well as by backlogging orders (i.e., \$1 per case of beer per week) and are assessed at each stage of the supply chain (Sterman, 1989).

A model which is based on the Beer Distribution Game outlined by Sterman (1989) has already been developed by Kirkwood (1998) based on the software tool Vensim. The stock and flow structure of this model is depicted in Figure 1.

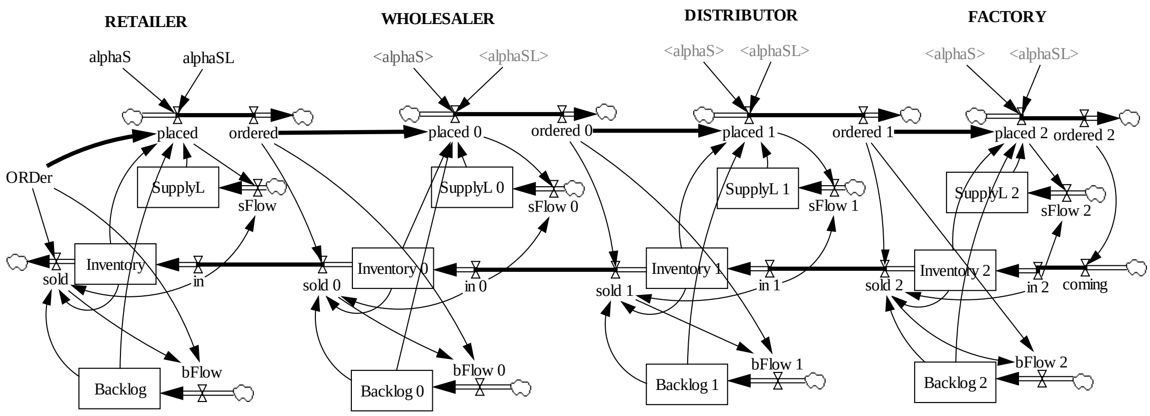


Figure 1: System Dynamics model of the Beer Distribution Game

To utilize this model for the testing of the proposed hypotheses, slight adjustments were necessary. For further analysis, the model was changed so that it incorporates the anchoring and adjustment heuristic listed in Equation 1 with the two proportional controllers α_{SL} and α_S. Additionally, the smoothing function of the forecasted expected loss rate was removed from the original model. Thus, the forecasted expected loss rate in the model used for investigation is solely based on the customer demand of the previous period, no smoothing method is applied. Therefore, Equation 2, which is used in the original model by Kirkwood (1998), was replaced by Equation 3 (because of the shorter acquisition lag of the factory, its order is only multiplied by 3 instead of 4). Besides these changes, the model remained unchanged in its basic structure.

Equation (2):
placed = MAX(0,SMOOTH(order,SMOOTHTIME)+A*(12-(Inventory-Backlog)-*SupplyL))

Equation (3):
placed = MAX(0,order+alphaS*(12-(Inventory-Backlog))+alphaSL*((order*4)-SupplyL))

Vensim's built-in optimization module, which employs Powell's conjugate direction method, was used to find the values of the proportional controllers α_S and α_{SL} that minimize the cost function. The model does not discriminate between different stages when it comes to the proportional controllers, meaning that the proportional controllers of the retailer are equal to the proportional controllers of the wholesaler, distributor and factory. This simplification is also used by other authors when it comes to the calculation of the proportional controllers which are not computed for each single stage but as a mean value over all stages (Croson et al., 2014; Sterman, 1989). To investigate Hypothesis 1, the constraint $\alpha_{SL} \leq \alpha_S$ was added and later, for the examination of Hypothesis 2, eliminated. This restriction was implemented in Vensim by introducing a variable which has a high value in case of $\alpha_{SL} > \alpha_S$ and a value of zero if $\alpha_{SL} \leq \alpha_S$ and directly affects the cost function. Because minimizing costs is the objective, this variable prevents cases of $\alpha_{SL} > \alpha_S$ as possible solutions as they result in prohibitively high costs.

The initial values of the variables and parameters are set according to the Beer Distribution Game described by Sterman (1989). The model is initialized in equilibrium with 12 units in every singles stage's inventory and 16 units in each stage's supply line (12 in case of the factory). The desired inventory is fixed as in the APIOBPCS theory with a value of 12 as hypothesized by Sterman (1989). The modelling of a flexible desired supply line level is also consistent with the APIOBPCS model; it is calculated by multiplying the lead time with the forecasted expected loss rate. Only the customer demand pattern traditionally used in the Beer Distribution Game (described above) was tested. The simulation length was set to 36 periods, corresponding to the game duration of real Beer Distribution Game runs (Sterman, 1989).

Simulation results and interpretation

First, Hypothesis 1 with the restriction $\alpha_{SL} \leq \alpha_S$ was tested. The optimization module computes an optimal value of 0.024 for both proportional controllers implying that the expected loss rate should be adjusted by 2.4 % of the discrepancy between desired and actual supply line as well as 2.4 % of the discrepancy between desired and actual stock. Because of the equality of the proportional controllers, Hypothesis 1 is supported. Consequently, if assuming an anchoring and adjustment heuristic following the DE-APIOBPCS (and therefore avoiding Supply Line Underweighting) leads to the lowest cost under the condition of $\alpha_{SL} \leq \alpha_S$. This can also be visualized by performing a sensitivity analysis. To that end, the proportional controller for the stock was kept constant at the calculated optimal level of 0.024. The proportional controller for the supply line was altered, from a minimum value of 0.001 to a maximum value of 0.024 with an increment of 0.001. Figure 2a shows the range of overall costs depending on the ratio of the two proportional controllers. Minimum costs associated with the DE-APIOBPCS policy, hence matching proportional controllers ($\alpha_{SL} = \alpha_S = 0.024$) are 581\$. The highest costs of 713\$ arise in case of maximum underweighting of the supply line ($\alpha_{SL} = 0.001$; $\alpha_S = 0.024$). It is noteworthy that overall costs develop relatively similar till around period 20, regardless of the ratio of the control parameters.

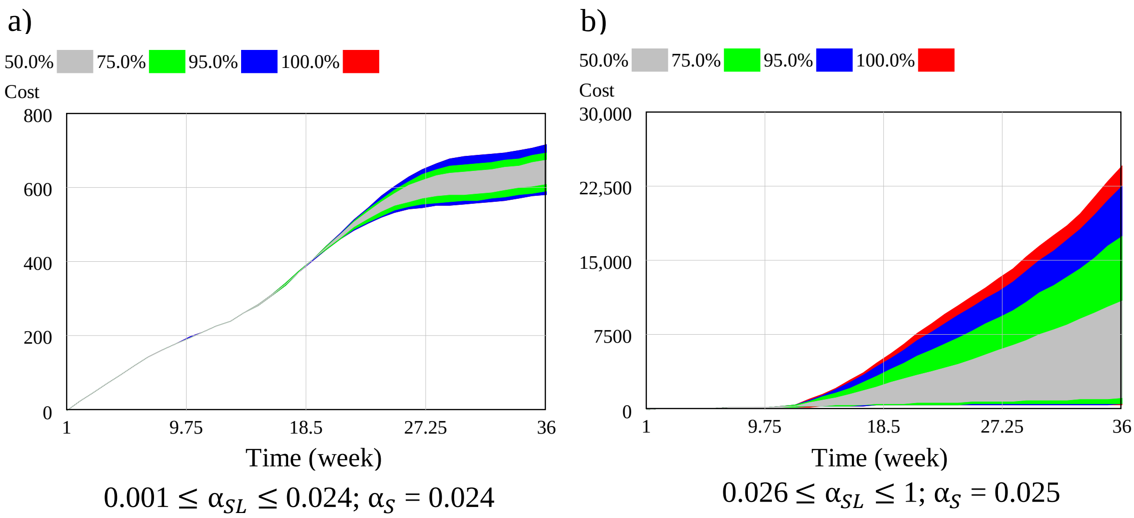
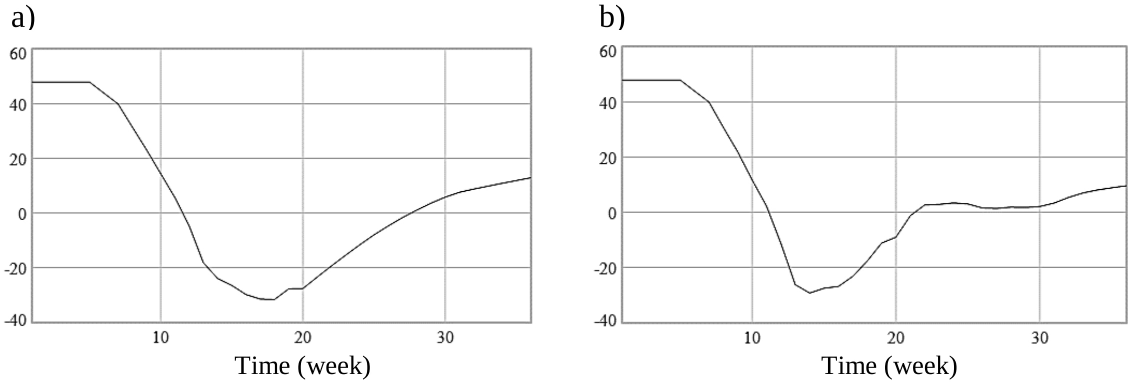


Figure 2: Sensitivity analysis

Both relevant types of costs, backlog and inventory costs, are reflected in the net inventory which is calculated by subtracting the backlog from the stage's inventory. If the net inventory is positive, the stage's inventory level is higher than its backlog. In case of a negative net inventory, it is vice versa. The sum of all stages' net inventories -the total net inventory- reflects whether the sum of all stages' inventories is higher or lower than the sum of their backlogs. During the first four periods of constant customer demand, the total net inventory remained in its initialized equilibrium of 48 units. The increase in demand disturbs the initial state of equilibrium and decreases the total net inventory due to the delay between the notification of a higher customer demand and the arrival of the replenishment order. Not only does this lead to depleted inventories throughout the supply chain but also to accumulated backlog, noticeable in the negativity of the total net inventory level (see Figure 3a).



$\alpha_{SL} = 0.024;\ \alpha_S = 0.024$ $\alpha_{SL} = 0.074;\ \alpha_S = 0.025$

Figure 3: Total net inventories of the optimized proportional controllers

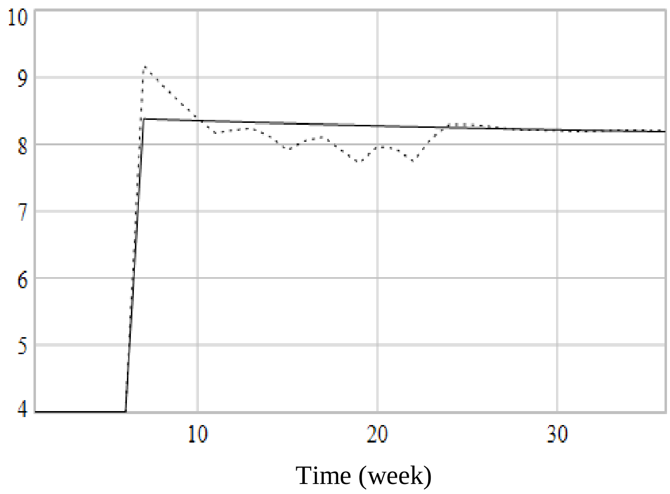
By analyzing the further progression of the total net inventory curve, a steady increase can be observed. The total net inventory in case of $\alpha_{SL} = \alpha_S =$ 0.024 turns positive again in week 28. The question remains if there is a combination of proportional controllers where the costs are even lower than in case of the optimally equal-weighted proportional controllers. Such a combination of proportional controllers could for example exist if it caused less backlog. At the same time, this combination must not produce higher costs for holding inventory than it saved for backlog costs. Because the range of $\alpha_{SL} \leq \alpha_S$ has already been covered, the potentially improved ratio of the proportional controllers must be -if even existent- in the area of $\alpha_{SL} > \alpha_S$. To cover this range and test Hypothesis 2, the restriction $\alpha_{SL} \leq \alpha_S$ is eliminated in the following analysis.

In case of abolishing the restriction $\alpha_{SL} \leq \alpha_S$, Powell's conjugate direction algorithm results in α_{SL} = 0.074 and $\alpha_S = 0.025$ as the values yielding the lowest costs after 36 simulated periods. Costs in this case are 475$, which is ~18% less than the costs achieved with the DE-APIOBPCS policy. Hypothesis 2 is supported. This combination of proportional controllers leads, according to the beforementioned criteria of α_{SL} / α_S > 0.5 to a stable system. This finding could also be confirmed through longer simulation durations (> 200 periods).

A sensitivity analysis provides clear evidence that overweighting the supply line can also be harmful (see Figure 2b). For this sensitivity analysis, α_S was kept constant at the calculated optimal value of 0.025 and α_{SL} varied from 0.026 to 1. The proportional controller for the supply line is limited to a value of 1 due to greater clarity of the figure. On the one hand, the sensitivity analysis in Figure 2b indicates a superiority by overweighting the supply line in some cases, but on the other hand clearly shows that most of the cases with a ratio of $\alpha_{SL} > \alpha_S$ result in very high costs. The highest cost of 24970$ is associated with the maximum overweighting allowed in this scenario which is $\alpha_{SL} = 1;\ \alpha_S = 0.025$. It is important to bear in mind that the sensitivity analysis in case of Supply Line Overweighting covers a much wider range of proportional controllers (0.026 ≤ α_{SL} ≤ 1; α_S = 0.025) than the sensitivity analysis in case of Supply Line Underweighting (0.001 ≤ α_{SL} ≤ 0.024; α_S = 0.024).

The comparison of total net inventories for the optimized proportional controllers with and without the restriction $\alpha_{SL} \leq \alpha_S$ (Figure 3a and Figure 3b) shows that the recovery to the area of positive total net inventory after the demand shock is faster in case of overweighting the supply line. In case of overweighting the overall net inventory turns positive again in period 22, whereas it is not before period 28 in case of matching proportional controllers. Figure 3 also shows that the total net inventory remains slightly above zero for an extensive time span in case of Supply Line Overweighting, whereas the optimally equal-weighted proportional controllers lead to a smooth but steady increase in the total net inventory.

As predicted by inventory and production control theory, a comparison of the order rates shows that there are no oscillations in case of equally weighted proportional controllers. However, in case of Supply Line Overweighting ($\alpha_{SL} = 0.074$; $\alpha_S = 0.025$) oscillations in the order rate exist. This is exemplarily depicted for the retailer's order rate in Figure 4.



—— $\alpha_{SL} = 0.024$; $\alpha_S = 0.024$ ------ $\alpha_{SL} = 0.074$; $\alpha_S = 0.025$

Figure 4: Order rates of the optimized proportional controllers

Conclusion

This study showed that investigating an imbalanced consideration of a supply line with an inventory and production control theory perspective can lead to several new insights. Using the Beer Distribution Game, simulations demonstrated an inferiority of Supply Line Underweighting in terms of costs incurred to the discussed DE-APIOBPCS policy, which refers to an equal consideration of both proportional controllers. After taking Supply Line Overweighting into consideration, it turned out that slightly overweighting the supply line is beneficial in the given setting. Furthermore, the study demonstrated that small changes of the decision makers' behavior can have a severe impact on the overall performance.

Assuming it was possible to set any value for the proportional controller α_S and let the players only decide about the proportional controller α_{SL}, the question arises if it would be better to either (1) provide the value of α_S = 0.025 and instruct individuals to overweight the supply line, or (2) to provide α_S = 0.024 and instruct them to equally take the proportional controller α_{SL} into account for their order decisions. One could argue that decision makers should be steered in the direction of overweighting because overweighting the supply line leads to the lowest costs in the simulation. However, the conducted sensitivity analysis clearly depicts that Supply Line Overweighting is only beneficial in very few cases regarding the studied demand pattern. Most often, Supply Line Overweighting leads to very high costs in the examined case. Another line of argumentation could be in favor of instructing decision makers towards an equally consideration of the proportional controllers, hence applying the DE-APIOBPCS policy. Even though this might not lead to the optimal performance, the DE-APIOBPCS has the great advantage to result in a stable system without oscillations in the order rate. This would also be beneficial for real companies because oscillating order rates inhibit efficient scheduling and production (Riddalls & Bennett, 2002).

By comparing the simulation results to empirically observed data it is notable that the calculated optimal value of the proportional controller α_{SL} in case of Supply Line Overweighting (α_{SL} = 0.074) is very close to the value Sterman (1989) derived after statistically analyzing several runs of the Beer Distribution game (α_{SL} = 0.088). This might indicate that decision makers' behavior regarding proportional controller α_{SL} is not as far off as one would initially assume. However, as the study showed, it is not a single proportional controller in isolation but the ratio of α_{SL} and α_S which determines the overall costs.

This study has several limitations. First, the system dynamics model used in the study assumes the same values for the proportional controllers throughout the supply chain. In real supply chains, the values of the proportional controllers probably differ from decision maker to decision maker. Second, the proportional controllers do not change during the simulation, whereas the proportional controllers could be dynamic. The system dynamics model allows for decimal digits, whereas in the real Beer Distribution Game decision makers can only place integer order quantities. Another simplification concerns the neglection of possible smoothing of the forecast. It is unlikely, that decision makers base their forecast of the customer demand solely on one period. In fact, it is more likely that they base their forecast on an (weighted) average of multiple previous periods.

Future studies could test different demand patterns about optimal proportional controllers. Here, question such as: "Does selecting equal proportional controllers always result in lower costs than proportional controllers indicating Supply Line Underweighting?" and "Do proportional controllers indicating Supply Line Overweighting always lead to lower costs than the optimally equal-weighted proportional controllers?" could be of interest. Another important issue to address is the influence that Supply Line Overweighting has on the bullwhip effect. Because of the advantages of equally weighted proportional controllers regarding stability, it could also be of interest to investigate how decision makers can be incentivized to equally weight the two controllers.

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