Designing the supplier-based revenue sharing contract for the goods experiencing inventory-dependent demand: A mathematical framework

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Abstract

This study addresses the mathematical design and formulation of a profit-sharing mechanism for supplier-based revenue sharing contract for the goods experiencing inventory-dependent demand to achieve an effective coordination between the supply chain (SC) partners. Both the non-coordinated and coordinated cases of supply chain network are discussed and compared based on their profit-sharing mechanisms, however, may be quite complex to be discussed. The insights from this study will construct a framework to formulate and develop an optimal ordering policy in anticipation to the overall cost reduction and profitability of the supply chain network as the objective functions.

Keywords: Demand rate function, Inventory-dependent demand, Supplier-based revenue sharing contract

Introduction

In contrast to the present scenario of ever changing global economy, managing the inventory or stock levels has become even more challenging due to drastic variations in demand rate for various products that has always kept material managers or supply chain (SC) professionals/practitioners in dilemma urging them to maintain how much of stock level. This study will confront that how an effective coordination can be achieved between the SC partners by mathematically designing and modelling a profit-sharing mechanism framework for supplier-based revenue sharing coordination policy subject to the condition: ‘the demand rate function depends on the stock or inventory level’.

Motivation behind the Study

The revenue sharing coordination policy that can also be conventionally termed as retailer-based revenue sharing coordination policy had been defined and framed by Cachon and Lariviere (2005) as; the retailer (buyer) pays a minimal amount for each unit purchased from the manufacturer/supplier (seller) but shares a fraction of its revenue to the manufacturer for each unit being sold. In contrast to this, this study has developed and designed a mathematical framework for the supplier-based revenue sharing coordination policy wherein, the supplier shares a fraction of its revenue to the retailer. As such, the manufacturer/supplier compensates for the retailer’s lost profit, and possibly, provides extra savings by offering the retailer a fraction of its revenue (ρ) for each number of units sold to the retailer. This will motivate the retailer to order more than its economic order quantity (EOQ) and will contribute towards an optimal ordering policy along with an effective profit-sharing mechanism.

Stock-Dependency Attribute of Demand Rate Function

In contrast to constant demand rate for particular items, the displayed inventory/stock level has a positive repercussion on the sales and profits. In many real-life situations, the demand rate gets influenced by the stock level like in case of some perishable goods (fruits, bread, dairy products, etc.) as such, these are to be sold out in a short time, and also, are produced in small quantities. With this type of product, the probability of making a sale would increase as the amount of the product in inventory increases. This instance very particularly indicates the variability of the demand rate with the inventory/stock level. Baker and Urban (1988) had conceptualized the inventory-dependency of demand rate function as the “inventory-dependent demand rate function”. These researchers had stated that the product’s demand rate would have to be presumed as the function of inventory/stock level i.e. the quantity on-hand with the underlying principle that there would be a hike in sales’ probability with an incremented value of the stock level.

Some of the literature citations to figure out the inventory-dependency attribute of demand rate function shown in the Table 1, give a considerable indication that the demand rate function has presumed to be as deterministic and positively inventory-dependent.

Table 1 - Literature citations for inventory-dependency attribute of demand rate function

Literature Studies	**Stock-Dependency Attribute of Demand Rate Function**
Baker and Urban (1988)	Polynomial Functional Form of Instantaneous Inventory Level
Giri and Chaudhuri (1998); Bhunia and Shaikh (2011); Omar and Zulkipli (2014)	Displayed Inventory/Stock Level
Urban (2005)	Both Initial Inventory Level and Instantaneous Inventory Level
Parthasarathi et al. (2010)	Initial Stock-Dependency of Short Life-Cycle Products with Random Demand
Yang et al. (2014); Parthasarathi et al. (2014)	The Retailer's Stock Level
Giri and Bardhan (2015)	On-Hand Stock Displayed at the Buyer’s End

Mathematical Significance of Stock-Dependency of Demand Rate Function

Towards this, Baker and Urban (1988) modelled a continuous and deterministic inventory system with the demand rate being dependent on the stock level. These researchers had worked on this model assuming instantaneous replenishments with a constant lead time, constant selling price and unit cost of the item with no price discounts and no effects of inflation, and constant procurement cost and holding cost allowing no backorders considering infinite time horizon for a single item having only one stocking point. As per the given model, the demand rate function for the product have a polynomial functional form for which there will have an incremental rise in the demand rate function with the inventory or stock level implying the stock-dependency of demand rate function. The stock-dependency of demand rate function from the said model had been mathematically expressed as;

$D(i_r) = a i_r^b$ (a > 0, 0 < b < 1) where,

i_r = Inventory/Stock Level,
a = Scale Parameter, and
b = Shape Parameter/Inventory-level Elasticity (Measured as the ratio of percentage change in the quantity being demanded to percentage change in the inventory level i.e. demand rate's responsiveness to change in inventory level, all other things to be equal)

The inventory function over time has been mathematically computed by equating the slope of curve at any point i.e. rate of change of inventory level per unit time with the negative value of demand rate function as the polynomial functional form of inventory level (i_r), and the solution to resulting differential equation has been given as;

$\frac{di_r}{dt} = -D(i_r) = -a i_r^b$ (a > 0, 0 < b < 1),

which on integrating both sides, we get,

$\int i_r^{-b} di_r = \int -a dt,$ which gives,

$$\frac{i_r^{1-b}}{1-b} = -at + c$$

$$or\ \ i_r^{1-b} = -[a(1-b)t] + k \ \ \text{[Taking } c\,(1-b) = k]$$

When t = 0, $i_r = Q_0$, so that

$$k = Q_0^{1-b}$$

and $i_r^{1-b} = Q_0^{1-b} - [a(1-b)t]$ ------- (1)

This will generate the value of inventory/stock level, i_r on solving as such;

$$i_r = \begin{cases} \left(Q_0^{1-b} - [a(1-b)t]\right)^{1/_{1-b}} & for\ \ t \le \dfrac{Q_0^{1-b}}{a(1-b)} \\ 0, & otherwise \end{cases}$$

As the stock/inventory level at the retailer's end becomes zero, this will deduce the replenishment cycle length as,

$$T_{RC} = t = \frac{Q_0^{1-b}}{a(1-b)} \text{ ------- } (2)$$

Impact of Stock (Inventory) Level on Demand Rate Function

Significantly, several research studies had been carried out to assess that how the shelf-space allocation gets affected with the retail-product demand taking into account the products facing such type of demand. Initially, Whitin (1957) established and addressed the inter-dependency of sales and inventory with each other by presuming that the higher level of inventory may lead to marginal growth in the sales.

Both Levin et al. (1972), and Silver and Peterson (1985) stated the proportionality of the consumption rate to the stock level being displayed. Levin et al. (1972) had referred to one of the functions of stock/inventory level as that of a motivator, as such;

'At times, the presence of inventory has a motivational effect on the people around it. It is a common belief that large piles of goods displayed in a supermarket will lead the customer to buy more'

To assess the stock-dependency of demand rate function, apart from all these studies, various other researchers likewise, Abbott and Palekar (2008), Baker and Urban (1988), Chang et al. (2006), Chang et al. (2010), Goyal and Chang (2009), Goswami and Chaudhuri (1992), Mandal and Phaujdar (1989), Padmanabhan and Vrat (1995), Pal et al. (1993), Phelps (1980), Ritchie and Tsado (1985), Sana (2010, 2011a, 2011b and 2012), Sarkar et al. (2010), Silver and Meal (1979), Silver and Peterson (1985) and Urban (1992) also had reviewed about the significance of stock-dependent demand rate function.

Integrating Stock-dependent Demand Rate Function with SC Coordination Policies

With increasing emphasis on the significance of effective SC coordination, many of the researchers like such that Monahan (1984), Weng (1995), Raju and Zhang (2005), and Li and Liu (2006) have devoted considerable attention to the coordination issues between manufacturers (suppliers) and retailers (buyers) in a SC network. Most of them, however, assumed that the supply chain's market demand was either price-sensitive or constant. As a matter of fact, the demand rate function is usually influenced by many factors such as stock/inventory level, service, advertisement, price, etc.

Although many of the authors incorporated the inventory-dependency attribute of demand rate function as discussed previously in the literature studies into inventory models but not many of them discussed about the coordination issues in the SC network integrating with the inventory-dependency of demand rate function.

Likewise, Hariga and Al-Ahmari (2013) designed the profit-maximization models for a single retail item facing the stock-dependent demand by mathematically integrating the retail shelf space allocation with inventory models for a two-stage supply chain functioning under vendor-managed inventory (VMI) along with consignment stock (CS) policies to economically correlate the supplier–retailer relationships (with and without partnership).

While Yang, Hong and Lee (2014) mathematically, formulated models for three of the coordination approaches: credit period policy, quantity discount policy and centralized SC policy taking into account the stock-dependency attribute of demand rate function from Baker and Urban (1988) such that the demand rate function equalizes the reduction in the inventory/stock level wherein the depletion of inventory level at the retailer's side takes place at a diminishing rate as far as the stock level tends to zero. They compared the total profits under first two policies, and then, extended to a centralized SC policy to validate an order quantity to be distinct and optimal to perfectly coordinate a supply chain featuring of a single manufacturer and a retailer with a single item.

Zhou, Min and Goyal (2008) also designed the framework for a decentralized two-echelon SC comprising a single manufacturer and a retailer for a single item in accordance with the stock-dependent demand function as deduced from Baker and Urban (1988) by validating numerically. They worked out on three major issues- firstly, manufacturer-Stackelberg game network for the assessment of deciding criteria for the item's wholesale price by the manufacturer followed by estimation of order quantity by the retailer and secondly, formulation of a non-complex profit-sharing (PS) mechanism thereon, to achieve perfect coordination within the SC network.

Working on the same demand model by Baker and Urban (1988), Giri and Bardhan (2015) modelled a single vendor-buyer two-layer SC to get the optimal solution for delivery batch size i.e. length of replenishment period and the number of shipments in order to reduce the average cost for the system if both vendor and buyer abide by the consignment stock (CS) policy. They further elaborated this model in accordance with a realistic approach if the buyer would display the items on shelf space area of limited capacity.

Wang (2008) assessed the impact of coordination policy on SC overall profit within the SC network for deteriorating items with stock-dependent demand rate function as derived from Baker and Urban (1988) and thereby, formulated the mathematical model for optimal order quantity and ordering cycle considering two different cases- decentralized (non-coordinated) SC policy and centralized (coordinated) SC policy and then, numerically validated the said model.

Problem Statement and Mathematical Assumptions

The profit-sharing mechanism for supplier-based revenue sharing coordination policy has been mathematically formulated and designed, and compared with the non-coordinated case if the demand rate function is to be totally inventory/stock level-dependent.

Mathematical Assumptions for the Proposed SC Coordination Mechanism

These mathematical assumptions have been considered while designing the mathematical framework for the SC coordination mechanism taking into account the stock-dependency attribute of demand rate function towards an optimal ordering policy:

1) The demand experienced by the retailer will be instantaneous inventory-dependent that had been mathematically deduced from Baker and Urban (1988) as such, stock-dependent demand rate function, $D(t) = a\, i_r(t)^b$, for $a > 0$, $1 > b > 0$

2) The manufacturer will adapt the lot-for-lot policy.

3) If all the goods are bought out, the inventory has to be replenished at the retailer's end.

4) Transportation cost incurred by the manufacturer$= (C_T Q_0 + C_{FS})$ where, $C_T =$ unit cost *of* transportation, and C_{FS} = fixed cost/shipment

5) Lead time will be zero, and no Shortages are allowed.

Under the consideration of stock-dependency attribute of demand rate function from Baker and Urban (1988), the profit-sharing mechanism for the non-coordinated SC network has been modelled and discussed and, then, thereafter, similar proposed mechanism for the supplier-based revenue sharing coordination policy has been mathematically formulated for a single-stage supply chain that will encompass a single manufacturer and a single retailer. The manufacturer will produce goods at a steady production rate R within each replenishment cycle for manufacturer's production length per cycle T_M for which $T_M \leq T_{RC}$ (where, T_{RC} = length of replenishment cycle) and will deliver those goods to the retailer, as each of the cycle ends. The stock/inventory level at the retailer's end will diminish at a reducing rate due to the inventory-dependency of demand rate function, as far as the stock level tends to zero. As such, the demand rate equalizes the reduction in the inventory level, the stock level at the retailer's end $i_r(t)$ will be characterized by this differential equation as;

$$\frac{di_r(t)}{dt} = -ai_r(t)^b$$

Notations

S = Selling price per unit, W = Wholesale price per unit, Q_O = Order quantity (decision variable), C_M = Manufacturer's production cost per unit, $i_r(t)$ = Retailer's stock level at time t, R_M = Manufacturer's production rate, C_{HR} = Holding cost per unit per unit time for the retailer, C_{HM} =Holding cost per unit per unit time for the manufacturer, T_{RC} = Replenishment Cycle Length, T_M = Manufacturer's production length per cycle, C_{OR}= Retailer's ordering cost, C_{OM} = Manufacturer's set up cost, π_R= Retailer's average profit, π_M = Manufacturer's average profit, ρ = A fraction of revenue shared by the manufacturer/supplier (seller) to the retailer (buyer) for each number of units sold

Non-coordinated Case

The retailer's economic order quantity (EOQ) and manufacturer's economic production quantity (EPQ) and their profits are to be mathematically deduced, in case the SC partners are not coordinating between themselves.

The cost components of retailer's profit function π_R^{NC} are sales revenue (selling price per unit), purchasing cost (wholesale price per unit), ordering cost and holding cost. Accordingly to this non-coordinated case, the retailer's profit function π_R^{NC} as the objective function may be mathematically expressed as;

$$\pi_R^{NC} = \frac{I}{T_{RC}} \left[(S-W)Q_0 - C_{OR} - C_{HR} \int_0^{T_{RC}} i_r(t)dt\right]$$
$$= \frac{1}{T_{RC}} \left[(S-W)Q_0 - C_{OR} - C_{HR} \int_0^{T_{RC}} \{Q_0^{1-b} - a(1-b)t\}^{1/_{1-b}} dt\right] \text{ ----- (3)}$$

[Using the value of i_r(t) from eq^n. (1) given by Baker and Urban (1988)]

Applying the value of $T_{RC} = \frac{Q_0^{1-b}}{a\,(1-b)}$ from eq^n. (2), as inferred from Baker and Urban (1988) into eq^n. (3), then, the second derivative $\frac{d^2\pi_R^{NC}}{dQ_0^2}$ deduces a negative term i.e. less than zero which means that π_R^{NC} is concave in Q and gives the value of retailer's optimal EOQ Q_0^* from $\frac{d\pi_R^{NC}}{dQ_0} = 0$ and with substituting this value of retailer's optimal EOQ Q_0^* in eq^n. (3) of π_R^{NC}, this deduces the retailer's optimal average profit π_R^{NC*} as the objective function.

The cost components of manufacturer's profit function π_M^{NC} are sales revenue (selling price per unit), production cost (manufacturer's production cost per unit), set up cost, transportation cost and holding cost. In case of no coordination, the manufacturer will have to proceed further keeping in view the retailer's decision where, the mathematical expression for manufacturer's profit function π_R^{NC} as the objective function is given as;

$$\pi_M^{NC} = \frac{I}{T_{RC}} \left[(W - C_M)Q_0 - C_{OM} - (C_T Q_0 + C_{FS}) - \frac{1}{2} C_{HM}\, Q_0 T_M\right] \text{ ------ (4)}$$

On replacing Q_0 by Q_0^* , the manufacturer's optimal average profit π_M^{NC*}can be deduced, in spite of this, the retailer's optimal EOQ Q_0^* generally varies from the manufacturer's optimal EPQ Q_0^{**}, which has more often higher value than the retailer's optimal EOQ Q_0^*.

As $T_M = \frac{Q_0}{R_M}$ and substituting this into eq^n. (4),

$$\pi_M^{NC} = \frac{I}{T_{RC}} [(W - C_M)Q_0 - C_{OM} - (C_T Q_0 + C_{FS}) - \frac{1}{2} C_{HM} \frac{Q_0^2}{R_M} \text{ ------ (5)}$$

Applying the value of $T_{RC} = \frac{Q_0^{1-b}}{a\,(1-b)}$ from eq^n. (2), as given by Baker and Urban (1988) into eq^n. (5), then, the second derivative $\frac{d^2\pi_M^{NC}}{dQ_0^2}$ deduces a negative term i.e. less than zero which means that π_M^{NC} is concave in Q and gives the value of manufacturer's optimal EPQ Q_0^{**} from $\frac{d\pi_M^{NC}}{dQ_0} = 0$ and with substituting this value of manufacturer's optimal EPQ Q_0^{**} in eq^n. (5) of π_M^{NC}, this deduces the manufacturer's optimal average profit π_M^{NC*}as the objective function.

Interpretation from Non-coordinated Case
$\boldsymbol{\pi_M^{NC} > \pi_R^{NC}}$, which indicates the higher profit for manufacturer for which the retailer has to order more and the manufacturer's optimal EPQ will have higher values than the retailer's optimal EOQ i.e. $Q_0^{**} > Q_0^*$

Supplier-based Revenue Sharing Coordination Policy (Coordinated Case)
When the retailer's order quantity is larger than its EOQ, the manufacturer offers the retailer a fraction of its revenue (ρ) for each number of units sold which motivates the retailer to increase its order quantity as a profit-sharing mechanism.

The manufacturer's average profit function as the objective function for this coordination policy comprising of the cost components - sales revenue (selling price per unit), production cost (manufacturer's production cost per unit), set up cost, transportation cost and holding cost, has been mathematically deduced as such;

$$\pi_M^{RS} = \frac{1}{T_{RC}} \left[(W - C_M)Q_0 - C_{OM} - (C_T Q_0 + C_{FS}) - \frac{1}{2R_M} C_{HM} {Q_0}^2 - \rho W Q_0\right] \text{ ------ (6)}$$

(where, manufacturer's revenue = WQ_0)

subject to

$$\pi_R^{RS} - \pi_R^{NC*} = \frac{1}{T_{RC}} [(S - W)Q_O - C_{OR} - C_{HR} \int_0^{T_{RC}} i_r(t)dt + \rho W Q_0] - \pi_R^{NC*} \geq \Delta\pi_R \text{ ----- (7)}$$

The said constraint condition from eq^n. (7) implies that the retailer either gets equal or more profit in compare to the optimal profit for the non-coordinated case.

Note: There will be an optimal solution Q_0^{**} that will be distinct in nature at which π_M^{RS}will have a maximum value for $\pi_R^{RS} - \pi_R^{NC*} = 0$

Using the eq^n. (7), $\pi_R^{RS} - \pi_R^{NC*} = \Delta\pi_R$, we can get the value of ρ as the function of Q_0 from $\pi_R^{RS} - \pi_R^{NC*} = 0$ as given;

$$\rho\,(Q_0) = \frac{1}{WQ_0} T_{RC}\,(\pi_R^{NC*} + \Delta\pi_R) - \frac{1}{W}\,(S - W) + \frac{1}{WQ_0} C_{OR} + \frac{1}{WQ_0} C_{HR} \int_0^{T_{RC}} i_r(t)dt \text{ ----- (8)}$$

Substituting the value of ρ from eq^n. (8) into eq^n. (6), we can determine the manufacturer's profit function as such;

$$\pi_M^{RS} = a\,(1-b)\,[(S - C_M - C_T\,)Q_0^b - (C_{OM} + C_{FS})\,Q_0^{b-1} - \frac{C_{HM}}{2R_M}\,Q_0^{1+b} - \frac{\pi_R^{NC*} + \Delta\pi_R}{a\,(1-b)} - C_{OR}\,Q_0^{b-1} - \frac{C_{HR}}{a\,(2-b)} Q_0]\text{-----(9)}$$

Taking the 1st and 2nd derivatives of π_M^{RS} w.r.t. Q_0 in eq^n. (9), we can get optimal Q_0^{**} by equating as, $\frac{d\pi_M^{RS}}{dQ_0} = 0$.

Since,

$$\frac{d\pi_M^{RS}}{dQ_0} = a(1-b)[b\,(S - C_M - C_{FS})\,Q^{b-1} - (C_{OM} + C_{OR} + C_{FS})(b-1)Q_0^{b-2} - \frac{C_{HM}}{2R_M}\,(b+1)Q_0^b - \frac{C_{HR}}{a\,(2-b)}]$$

or $$\frac{d^2\pi_M^{RS}}{dQ_0^2} = a(1-b)[b(b-1)(S - C_M - C_{FS})Q_0^{b-2} - (b-1)(b-2)(C_{OM} + C_{OR} + C_{FS})Q_0^{b-3} - \frac{C_{HM}}{2R_M}\,b(b+1)Q_0^{b-1}]$$

$$\frac{d^2\pi_M^{RS}}{dQ_0^2} = -a(1-b)[b(1-b)(S - C_M - C_{FS})Q_0^{b-2} + (1-b)(2-b)(C_{OM} + C_{OR} + C_{FS})Q_0^{b-3} + \frac{C_{HM}}{2R_M}\,b(b+1)Q_0^{b-1}] < 0\text{-----(10)}$$

If π_M^{RS} is concave in Q_0, we can get the manufacturer's optimal EPQ Q_0^{**} from eq^n. (10) at $\frac{d\pi_M^{RS}}{dQ_0} = 0$.

Then, by substituting Q_0^{**} into π_M^{RS}, we can get the manufacturer's optimal average profit π_M^{RS*} as the objective function.

Concluding Remarks

In the supplier-based revenue sharing contract, there will be higher profit margin for the manufacturer if the retailer still holds the same profit as in the non-coordinated case irrespective of which, the profit-sharing for the retailer will not be satisfactory. Such circumstance may necessitate and ask for a negotiation of the retailer with the manufacturer for a higher profit margin. Both the non-coordinated case and the coordinated case (supplier-based revenue sharing policy) are absolutely complex to be discussed as such, if the order quantities have not the same values, the demand rates at the retailer's end also get varied due to the inventory-dependency of demand rate function.

The insights from this study will construct a conceptual and mathematical framework towards an effective profit-sharing coordination mechanism within the SC network and develop an optimal ordering policy for the consumer goods experiencing the inventory-dependent demand in anticipation to the overall cost reduction and the profitability of the SC network as the objective functions. This study can be further extended such that the stock-dependent deterministic demand can be advanced to the stochastic demand.

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